AD 700 963

ERRORS IN THE TRAIL RESULTING FROM IGNORING EITHER THE MEASURED TIME OF FLIGHT OR THE MEASURED RANGE

R. H. Kent, et al

Ballistic Research Laboratories
Aberdeen Proving Ground, Maryland

20 April 1939

AD 700963

BRL R 143

BRL

AD

REPORT NO. 143

ERRORS IN THE TRAIL RESULTING FROM IGNORING EITHER THE MEASURED TIME OF FLIGHT OR THE MEASURED RANGE

by

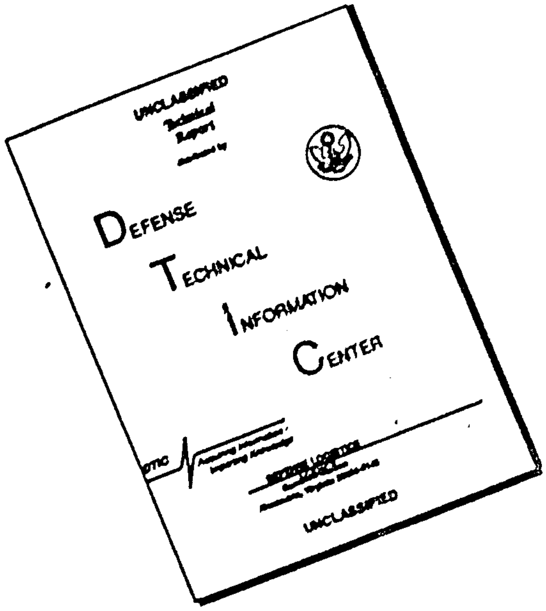
R. H. Kent
F. V. Reno

April 1939

U.S. ARMY ABERDEEN RESEARCH AND DEVELOPMENT CENTER
BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

16

DISCLAIMER NOTICE



THIS DOCUMENT IS BEST QUALITY AVAILABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

MAY 10 1939

Ballistic Research
Laboratory Report No. 143

RHK/FVR/emh
Aberdeen Proving Ground, Md.
April 20, 1939

ERRORS IN THE TRAIL RESULTING FROM IGNORING EITHER THE MEASURED TIME OF FLIGHT OR THE MEASURED RANGE

Abstract

Expressions for the errors in the trail, range, deflection and time of flight resulting from ignoring either the measured time of flight or the measured range are deduced. Results of calculations of the resulting errors in trail, time of flight, range and deflection for the bomb M38A2 are presented. The desirability of the introduction of a ballistic coefficient, C, based on trail, is mentioned. The reality and the sources of the discrepancy between the ballistic coefficients deduced from range and time of flight respectively are discussed.

Two different procedures have been used to determine the trail of a bomb from observations on certain elements of its trajectory. One of these procedures, designated by Procedure A, uses measurements of the range and ignores the measured time of flight. The other procedure, designated by B, uses measurements of the time of flight and ignores the range. In the following we shall not attempt to describe the complicated methods actually used but for the sake of clarity and brevity shall describe other procedures theoretically simpler which might be used for the determination of the trail from observations on the range or the time of flight.*

* Although the methods to be described are simpler in theory than those actually used, they are much more laborious in practice.

We adopt a coordinate system fixed with respect to the earth with origin at the vertical projection of the airplane on the ground and with the vector ground speed as the X axis. (See Fig. 1).

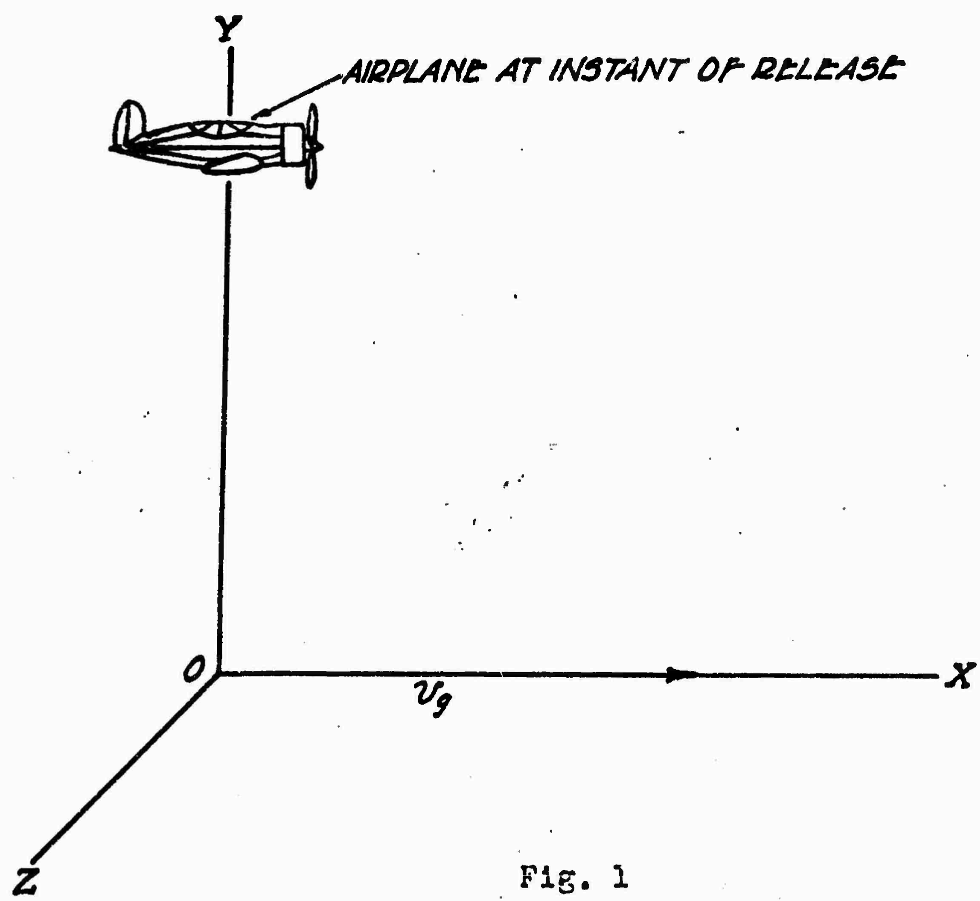


Fig. 1

We adopt symbols as follows.

C_x = ballistic coefficient determined from x .

C_t = ballistic coefficient determined from t .

t_ω = time of flight under ballistic table conditions.

u = resultant air speed.

u_x = range component of air speed.

u_z = deflection component of air speed.

v_g = ground speed.

x_ω = range under ballistic table conditions.

$x_{\omega b}$ = range under bombing table conditions.

$x_{\omega A}$ = range under bombing table conditions using data obtained by Procedure A.

$x_{\omega B}$ = range under bombing tables conditions using data obtained by Procedure B.

y_ω = altitude of airplane at release.

$z_{\omega b}$ = deflection under bombing table conditions.

$z_{\omega A}$ = deflection under bombing table conditions using data obtained by Procedure A.

$Z_{\omega B}$ = deflection under bombing table conditions using data obtained by Procedure B.

Δt = error in t_ω .

Δx = error in x_ω .

Δz = error in $z_{\omega b}$.

$\Delta\lambda$ = error in λ .

λ = trail under bombing table conditions.

λ_A = trail under bombing table conditions obtained from data of Procedure A.

λ_B = trail under bombing table conditions obtained from data of Procedure B.

λ_x = x component or range component of λ .

λ_z = z component or deflection component of λ .

λ_{xA} = x component of λ_A.

λ_{zA} = z component of λ_A.

λ_{xB} = x component of λ_B.

λ_{zB} = z component of λ_B.

Procedure A

The procedure is described in the following. The range is obtained from direct measurements of the position of the airplane at the instant of release and the coordinates of the point of fall taken with respect to the earth. The range obtained in this way is then corrected to ballistic table conditions. These latter assume no wind, standard density and temperature structure and no rotation of the earth. The range corrected to these ballistic table conditions is designated by x_ω . The ballistic coefficient is determined from x_ω and is designated by C_x. The time of flight is now computed from C_x and is designated by $t_\omega(C_x)$. The trail λ_A is then computed by the relation

$$\lambda_A = v_g t_\omega(C_x) - x_\omega = u t_\omega(C_x) - x_\omega . \qquad (1)$$

Since there is no wind, $u = v_g$ and $\lambda_A = \lambda_{xA}$.

In general, however, $t_\omega \neq t_\omega(C_x)$

since the ballistic coefficient determined from range C_x differs from that determined by the time of flight, C_t.

Let $\Delta t = t_\omega(C_x) - t_\omega$ and let $\Delta\lambda$ = the error in the trail.

Since the correct trail under both bombing table and ballistic table conditions is given by

$$\lambda = u\, t_\omega - x_\omega$$

it follows that,

$$\Delta\lambda = \lambda_A - \lambda = u\, t_\omega(C_x) - u t_\omega = u\, \Delta t. \qquad (2)$$

Suppose that bombs are subsequently dropped, under bombing table conditions with λ_A and $t\,(C_x)$ as determined by Procedure A. The bombing table conditions assume a standard density and temperature structure and no rotation of the earth. The wind is constant in intensity and direction but not necessarily zero. The air speed, u, is assumed to be equal to that obtained when the trail λ_A was determined and of course the altitude y_ω is the same. The air speed, u,

makes an angle, φ, with the ground speed v_g as shown in Fig. 2. The angle φ is the azimuth of the trail and under the stated conditions is equal to ψ the drift angle. The correct range under standard bombing conditions, $x_{\omega b}$, is given by

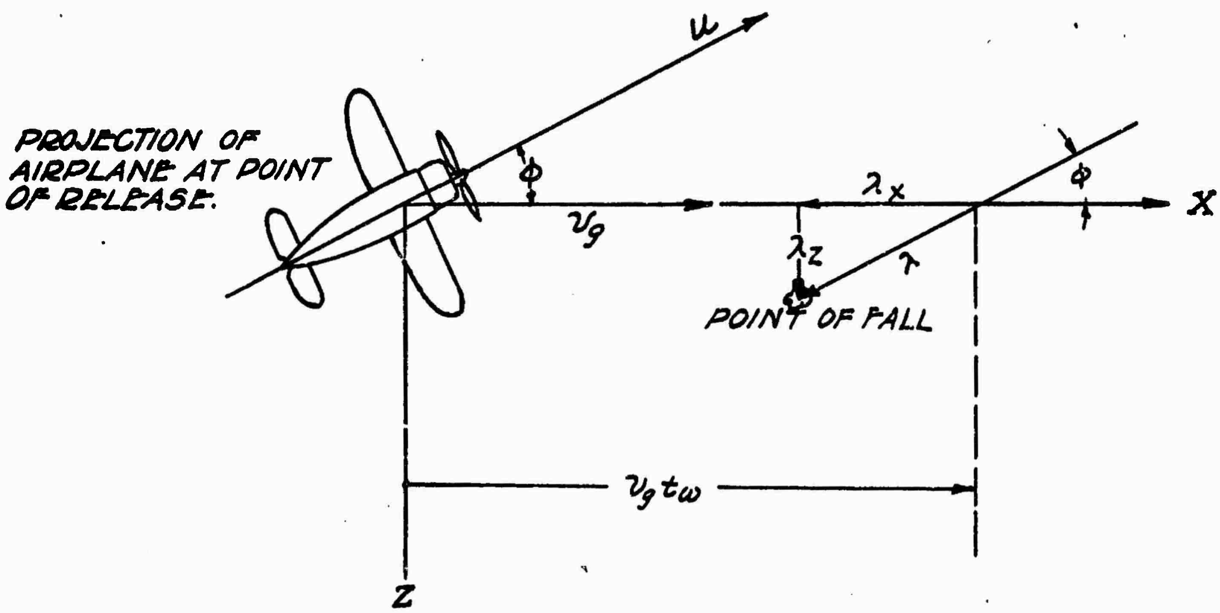


Fig. 2

$$x_{\omega b} = v_g t_\omega - \lambda \cos \varphi$$

while the range, $x_{\omega A}$, obtained by using λ_A and $t_\omega(C_x)$ will be

$$x_{\omega A} = v_g t_\omega(C_x) - \lambda_A \cos\varphi = v_g(t_\omega + \Delta t) - (\lambda + u\Delta t)\cos\varphi$$

from (2).

Hence the error in the range, Δx, from using Procedure A is

$$\Delta x = x_{\omega A} - x_{\omega b} = v_g(t_\omega + \Delta t - t_\omega) - (\lambda + u\Delta t - \lambda)\cos\varphi$$

$$= v_g\Delta t - u\Delta t\cos\varphi = \Delta t(v_g - u_x) = \Delta t w_x$$

where u_x is the x component of u and w_x is the x component of the constant wind. If the ground speed is equal to the air speed component, u_x,

$$\Delta x = 0.$$

In other words although Procedure A produces errors both in t_ω and λ, the resultant range will be correct unless

$$v_g \neq u_x.$$

The deflection under bombing table conditions, $z_{\omega b}$, is given by

$$\lambda \sin\varphi$$

while according to Procedure A the deflection would be

$$z_{\omega A} = \lambda_A \sin\varphi = (\lambda + u\Delta t)\sin\varphi.$$

Hence the error in the deflection, Δz, is

$$\Delta z = (\lambda + u\Delta t)\sin\varphi - \lambda\sin\varphi = u\Delta t \sin\varphi = u_z\Delta t = -w_z\Delta t$$

where u_z is the z component of u and w_z is the z component of the constant wind.(see Fig. 3).

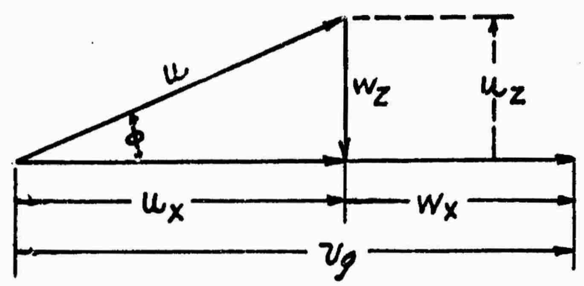


Fig. 3

The deflection obtained by Procedure A will be in error unless $\Delta t = 0$.

Procedure B

Under Procedure B, the only observationally measured element employed in obtaining the trail is the time of flight, reduced to ballistic table conditions. The ballistic coefficient, C_t, is calculated directly from the reduced time of flight. It is then assumed that the range is the tabular range x (C_t) corresponding to C_t, and the trail, λ_B, is obtained by the relation

$$\lambda_B = v_g t_\omega - x_\omega (C_t)$$

The correct trail under the ballistic table conditions* for which the range $x(C_t)$ is computed, is given by

$$\lambda = v_g t_\omega - x_\omega .$$

Hence the error in the trail $\Delta\lambda$ is given by

$$\Delta\lambda = x_\omega - x_\omega(C_t) .$$

* Of course the magnitude of the trail under standard ballistic conditions is equal to the magnitude under standard bombing conditions since the magnitude of the trail depends upon the air speed not the ground speed.

Under bombing table conditions, Δx the error in the range is given by

$$\Delta x = \left[x_\omega(C_t) - x_\omega\right]\cos\varphi = \left[x_\omega(C_t)-x_\omega\right]\frac{v_g - w_x}{u} \quad .$$

The error in deflection obtained by Procedure B is given by

$$\Delta z = -\left[x_\omega(C_t) - x_\omega\right]\sin\varphi = \left[x_\omega(C_t) - x_\omega\right]\frac{w_z}{u} \quad .$$

A Correct Procedure

A correct procedure is of course to measure both t_ω and x_ω. The range and time of flight thus obtained can be reduced to the values which would have resulted under ballistic table conditions by removal of the effects on range and time of flight due to departures from standard ballistic table conditions. The trail under ballistic table conditions is accurately given by

$$\lambda = v_g t_\omega - x_\omega ,$$

where the values of x_ω and t_ω are the values of the measured range and time of flight reduced to ballistic table conditions.

The trail, in a perfectly general sense, is the perpendicular distance in the horizontal plane $y = 0$ from the point of impact to the vertical through the point vector $v_g t_\omega$. The definition arises from the phenomenon noted by the bombardier that the bomb in flight "trails" behind the vertical through the bomb rack.

In reducing the observations and preparing tables of the trail as a function of altitude it is sometimes convenient to introduce a coordinate system moving with the airplane as shown in Fig. 4. In this system, the trail is simply the distance from the Y axis to the point of fall. The initial velocity of the bomb is zero. At the airplane and, if bombing table conditions hold, at other points of the trajectory there is a wind in this coordinate system equal in magnitude to the airspeed of the airplane but opposite in direction. If bombing table conditions do not hold, the wind varies along the trajectory.

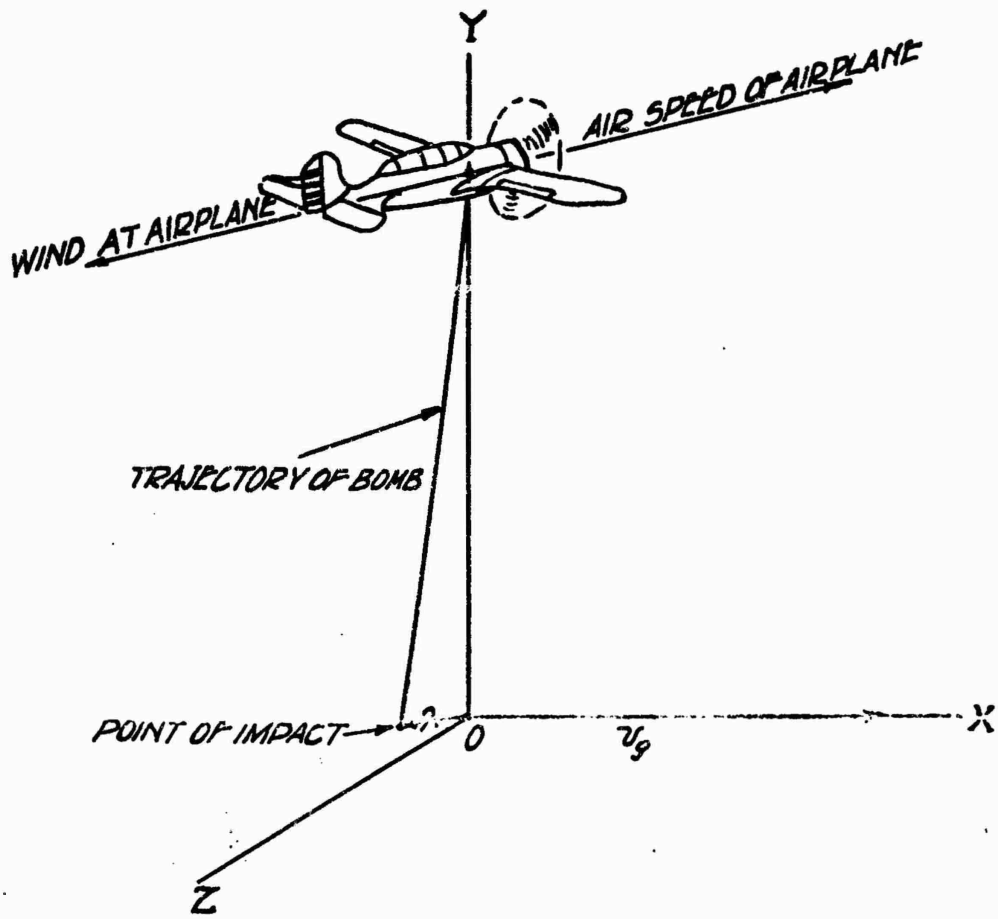


Fig. 4

Trajectories may be computed with such a coordinate system, the advantages of which will be discussed in a report to appear shortly. In such computations the ballistic coefficient which with the observed atmospheric structure produces a trail equal in magnitude and direction to the observed trail might be called the trail ballistic coefficient. However, in general it is impossible with a given C to reproduce exactly both the magnitude and direction of the trail. This results either from inaccurate measurements or an inadequate theory or a combination of the two. For practical purposes in the reduction of observations that ballistic coefficient which produces a trail under the observed meteorological structure and initial conditions equal in magnitude to the component of the observed trail in a direction determined by the wind structure is called the trail ballistic coefficient and is designated by C_λ .

A direct inference of C_λ can be made from a measured trail most conveniently from a table of λ as a function of u, y_ω and C. Such a table is being prepared and will be employed during the bomb test firing program at Aberdeen Proving Ground during the current year. In lieu of such a table, several equispaced values of λ as a function of u, y_ω and C can be constructed from existing tables of x_ω and t_ω from which the value of C_λ can be determined by inverse interpolation for any particular observed case. As a convenient and accurate approximation derivable from theory, accurate to within the individual probable error of one determination of C_λ it has been found that

$$C_\lambda = \frac{2C_xC_t}{C_x+C_t}.$$

The trail ballistic coefficient, once determined from observations, can be employed as argument in entering a table of trail angles in the construction of bombing tables, thus obviating the laborious construction of separate ranges and times of flight as functions of the C_x and C_t inferred from observation. A table of this kind, as one of a series of "Bomb Ballistic Auxiliary Tables" is being prepared, and the method is being currently employed in the production of bombing tables at Aberdeen Proving Ground.

A plot of C_λ vs y_ω for the bomb M38A2 is shown on plot 2.

NUMERICAL EXAMPLES

Table I shows the values of $\Delta\lambda$, Δt, Δx and Δz the errors under bombing table conditions in the trail, time of flight, range and deflection respectively for the bomb M38A2 for various conditions when the trail is determined by Procedure A. The values of C_t and C_x on which these results are based are given in plot 1.

Table I

Errors of Procedure A

Bomb M38A2, True Air Speed = 170 mi/hr

Altitude	Ground Speed mi/hr	Range Wind mi/hr	Cross Wind mi/hr	$\Delta\lambda$ ft	Δt sec.	Δx ft.	Δz ft.
5000	170	0	0	42	-0.17	0	0
	193.15	25	25	42	-0.17	6	6
15,000	170	0	0	57	-0.23	0	0
	193.15	25	25	57	-0.23	8	8
15,000	170	0	0	67	-0.27	0	0
	193.15	25	25	67	-0.27	10	10

Table II shows the errors under bombing table conditions in trail, $\Delta\lambda$, range, Δx, and deflection, Δz, resulting from the substitution of $x_\omega(C_t)$ for x_ω according to Procedure B.

Table II

Errors of Procedure B

Bomb M38 A2, True Airspeed = 170 mi/hr.

Altitude ft.	Ground Speed mi/hr.	Range Wind mi/hr.	Cross Wind mi/hr.	$\Delta\lambda$ ft.	Δx ft.	Δz ft.
5000	170	0	0	62	-62	0
	193.15	25	25	62	-61	9
10,000	170	0	0	72	-72	0
	193.15	25	25	72	-71	11
15,000	170	0	0	76	-76	0
	193.15	25	25	76	-75	11

Discrepancy between C_t and C_x, Sources of Discrepancy

It has been found by comparison of the experimental results for the firing programs of three bombs* at Aberdeen Proving Ground, that the difference between C_t and C_x is in general several times the experimental error.

In the reduction of the observations care has been taken to eliminate the important systematic errors. While it is realized that some systematic errors remain, it appears that their magnitude should be much too small to account for the observed discrepancy between C_t and C_x.** The problem of hitting a target can evidently be solved only if the currently tabulated trail angle, time of flight and dropping angle correspond to the considerably differing ballistic coefficients derived for these elements from experiment.

It appears that there are two possible sources of the discrepancy between C_t and C_x. In computing bomb trajectories, the bomb is currently assumed to be a particle and the drag function of that particle is assumed to be the Gâvre function, G_1. The shapes of bombs differ appreciably from the flat based projectile with an ogive of small radius on which the Gâvre function is based. Furthermore, the experimental difficulties in determining the drag at velocities below 900 ft/sec are considerable. For these reasons, it is likely that the drag function of the ordinary bomb differs appreciably from G_1. The use of an incorrect drag function would of course cause C_t to differ in general from C_x.

The bomb is not a particle. It has not only drag but also a cross wind force. These depend upon the angle of yaw. The neglect of this dependence will cause the calculated trajectories to differ from the observed ones and will produce in general a C_t different from C_x. In this connection mention is made of the following Ballistic Research Laboratory reports..

* Bomb, Practice, 100 lb. M38A2,
Bomb, Demolition, 1100 lb. M33.
Bomb, Demolition, 2000 lb. M34.

** The suggestion has been made that there may be a systematic plotting error. While there doubtless is one, it should not affect the measurement of the ground speed. The error in the position of the airplane caused by it is almost certainly far too small to account for the discrepancy between C_t and C_x.

No.	Title
X-45	Considerations Regarding the Flight of Vaned Projectiles
82	Effect of Yaw on Aircraft Bombs.

In the reports listed no account is taken of the initial angular velocity of yaw which results from the considerable angular velocity of the tangent to the trajectory.* Report No. 82 is now being revised by H. P. Hitchcock to include this effect.

Acknowledgments

The inaccuracies inherent in Procedures A and B for obtaining the trail were pointed out by Lt. Col. H. H. Zornig. This paper was written at his suggestion. Messrs. E. S. Martin and R. F. Cronin assisted in the preparation of the tables and plots.

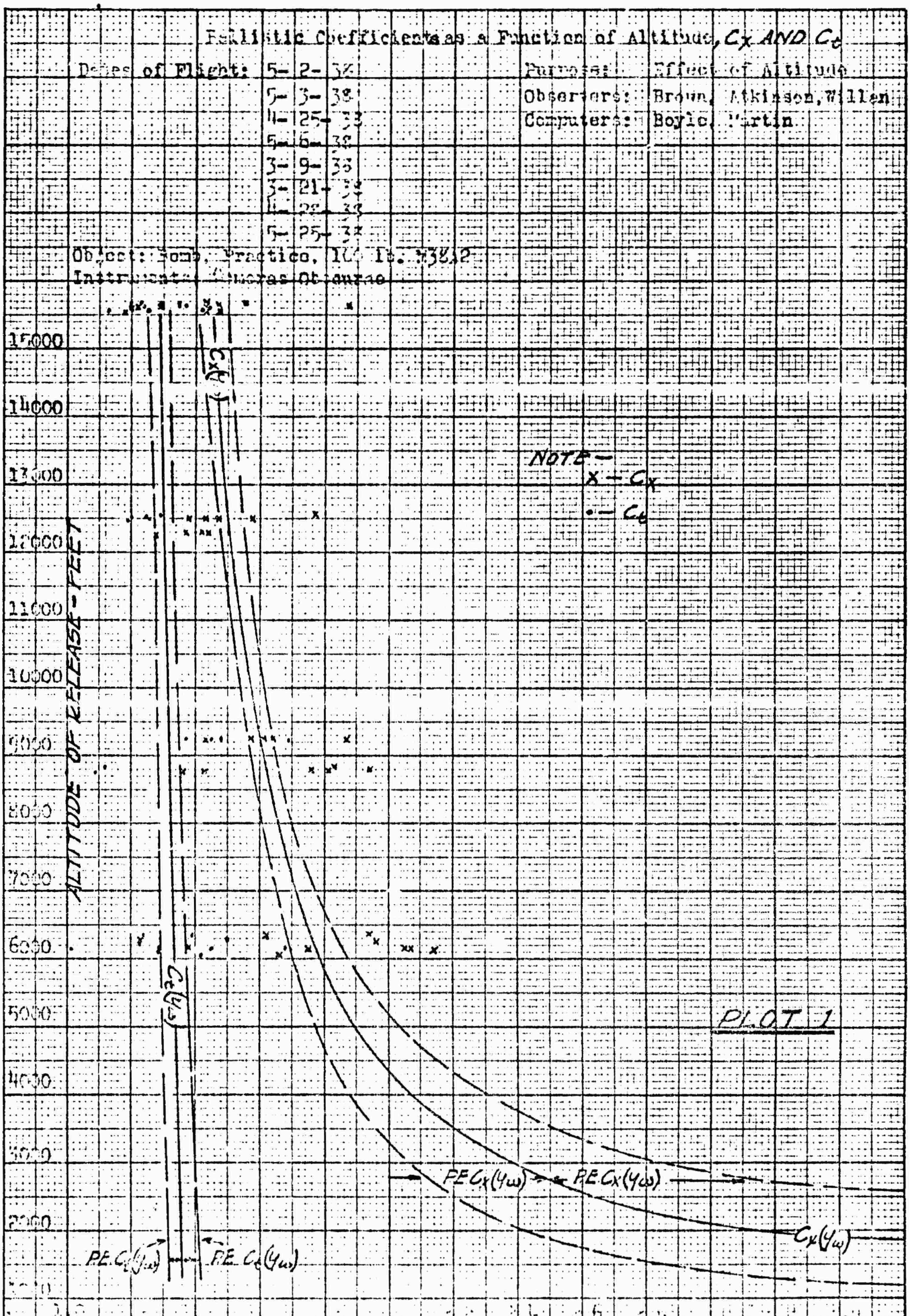
R. H. Kent

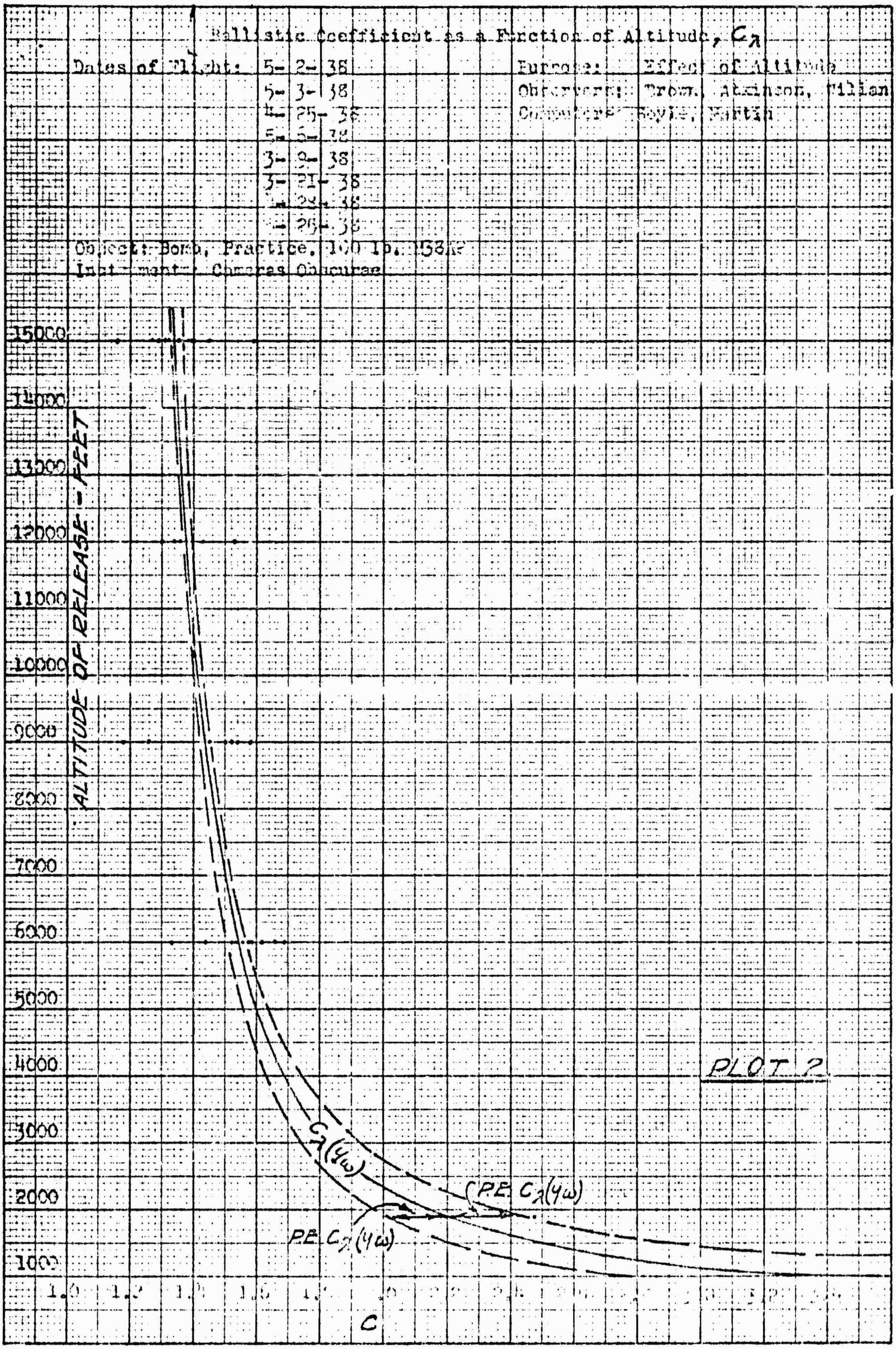
R. H. Kent

F. V. Reno

F. V. Reno

* See Darpas, Mem. d. l'Artilleries Franc. Tome. XVI, 4, p. 841, (1937). However, Darpas neglects the cross wind force entirely.


Ballistic Coefficients as a Function of Altitude, C_x AND C_e
Dates of Flight: 5-2-38
5-3-38
4-25-38
5-6-38
3-9-38
3-21-38
5-25-38
Purpose: Effect of Altitude
Observers: Brown, Atkinson, Willen
Computers: Boyle, Martin
Object: Bomb, Practice, 100 lb. M38A2
Instruments: Cameras Obscura
ALTITUDE OF RELEASE - FEET
15000
14000
13000
12000
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
NOTE -
x — C_x
• — C_e
PLOT 1
$C_x(y_\omega)$
$C_e(y_\omega)$
P.E. $C_x(y_\omega)$
P.E. $C_e(y_\omega)$
.6
C


Ballistic Coefficient as a Function of Altitude, C_λ
Dates of Flight: 5- 2- 38
5- 3- 38
3- 9- 38
Purpose: Effect of Altitude
Object: Bomb, Practice, 100 lb.
Instrument: Cameras Obscuras
ALTITUDE OF RELEASE - FEET
15000
14000
13000
12000
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
1.0
1.2
1.4
1.6
C
$C_\lambda(4\omega)$
P.E. $C_\lambda(4\omega)$
P.E. $C_\lambda(4\omega)$
PLOT 2

TITLE: Errors in the Trail Resulting from Ignoring Either the Measured Time of Flight or the Measured Range AUTHOR(S): Kent, R. H.; Reno, F. V. ORIGINATING AGENCY: Aberdeen Proving Ground, Ballistic Research Lab., Aberdeen, (Md. PUBLISHED BY: (Same)	ATI- 42716 DIVISION (None) ORIG. AGENCY NO. BRL-R-143 PUBLISHING AGENCY NO. (Same)

DATE	DOC. CLASS.	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
April '39	Unclass.	U.S.	English	15	tables, diagr, graphs

ABSTRACT:

Expressions for the errors in trail, range, deflection and time of flight resulting from ignoring either the measured time of flight or the measured range are deduced. Results of calculations of the resulting errors in trail, time of flight, range and deflection for the bomb M38A2 are given. The desirability of the introduction of a ballistic coefficient, C , based on trail is mentioned. The reality and the sources of the discrepancy between the ballistic coefficients deduced from range and time of flight respectively are discussed.

DISTRIBUTION: Copies of this report obtainable from Central Air Documents Office; Attn: MCIDXD

DIVISION: Ordnance and Armament (22) SECTION: Ballistics (12) ATI SHEET NO.: R-22-12-39	SUBJECT HEADINGS: Bombing - Accuracy (16676.6)